B.Sc Part-II PAPER

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Ratio test If $\{a_n\}$ is a sequence such that $\underset{}{Lt}\ \frac{a_{n+1}}{a_n} = l$ where $|l| < 1$, then

$$\underset{n \to \infty}{Lt}\ a_n = 0$$

Proof- Since $\underset{n \to \infty}{Lt}\ \frac{a_{n+1}}{a_n} = l$, therefore corresponding to an arbitary positive quantity ε, an integer m can be found such that

$$\left| \frac{a_{n+1}}{a_n} - l \right| < \varepsilon \text{ for } n \geq m$$

$$\Rightarrow \left| \frac{a_{n+1}}{a_n} \right| < |l| + \varepsilon \text{ for } n \geq m$$

Again, since $|l| < 1$, we can find k such that $|l| + \varepsilon = k < 1$.

Thus $\left| \frac{a_{n+1}}{a_n} \right| < k$ for $n \geq m$

Hence putting $n = m, m+1, m+2$ ------. we have

$$\left|\frac{a_{m+1}}{a_m}\right| < k, \left|\frac{a_{m+2}}{a_{m+1}}\right| < k,$$

$$\left|\frac{a_{m+3}}{a_{m+2}}\right| < k, \quad \left|\frac{a_n}{a_{n+1}}\right| < k$$

Multiplying the above ratios we get.

$$\left|\frac{a_n}{a_m}\right| < k^{n-m} \Rightarrow |a_n| < k^n \left|\frac{a_m}{k^m}\right|$$

$$\Rightarrow |a_n| < ck^n, \text{ where } c = \left|\frac{a_m}{k^m}\right| \quad \text{—(1)}$$

Now, take the limit as $n \to \infty$

$\because c$ is finite and $k^n \to 0$ as $n \to \infty$ for k is a fixed number < 1,

therefore from (1), we get

$$\underset{n \to \infty}{Lt}\ a_n = 0.$$

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Applications

1) $\underset{n \to \infty}{Lt}\, a_n$ such that $a_n = n^p x^n$ where p is a positive number and $|x| < 1$.

Here $\dfrac{a_{n+1}}{a_n} = \left(\dfrac{n+1}{n}\right)^p x = \left(1 + \dfrac{1}{n}\right)^p \cdot x$

So that $\underset{n \to \infty}{Lt}\, \dfrac{a_{n+1}}{a_n} = x$.

$\therefore$ If $|x| < 1$, then $\underset{n \to \infty}{Lt}\, a_n = 0$

In particular, if $a_n = n x^n$, then for $|x| < 1$, $a_n \to 0$

Similarly if $a_n = n^2 x^n$, then for $|x| < 1$, $a_n \to 0$.

(ii) $\underset{n \to \infty}{Lt}\, a_n$ such that $a_n = \dfrac{x^n}{n!}$ where x is any real number.